Catalonia: Independence and Pensions: Technical Appendix*

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April 161, 2017

*We thank Juan Carlos Conesa for an early version of the code. We are also grateful to Jesús Díaz Díaz, who suggested that we should research this topic.

Contents

1 The Model Economy

We study an overlapping generations model economy with heterogeneous households, a representative firm, and a government, which we describe below.[1]

1.1 The Households

Households[2] in our model economy are heterogeneous and they differ in their age, $j \in J$; in their education, $h \in H$; in their employment status, $e \in \mathcal{E}$; in their assets, $a \in A$; in their pension rights, $b_t \in B_t$, and in their disability and retirement pensions, p_t^d and $p_t \in P_t$. Sets J, H, $\mathcal{E}$, A, B_t, and P_t are all finite sets which we describe below. We use $\mu_{j,h,e,a,b,p,t}$ to denote the measure of households of type (j, h, e, a, b, p) at period t. For convenience, whenever we integrate the measure of households over some dimension, we drop the corresponding subscript.

Age. Households enter the economy at age 20, the duration of their lifetimes is random, and they exit the economy at age 100 at the latest. Consequently, $J = \{20, 21, \ldots, 100\}$. Parameter ψ_{jt} denotes the conditional probability of surviving from age j to age $j+1$. This probability depends on the household's age and it varies with time, but it does not depend on the household's education level.

Fertility and immigration. In our model economy fertility rates and immigration flows are exogenous.

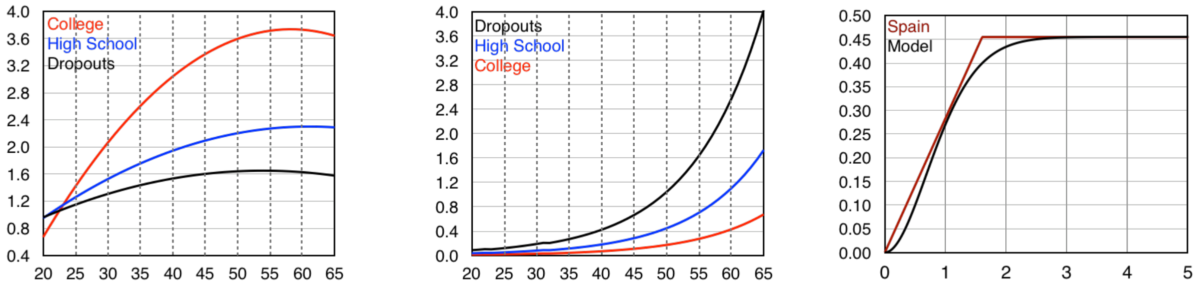
Education. Households can either be high school dropouts, high school graduates who have not completed college, or college graduates. The education level of a household that has dropped out of high school is $h=1$. The education level of a household that has has completed high school but has not completed college is $h=2$. And the education level of college graduates is $h=3$. Therefore, $H = \{1, 2, 3\}$. The education decision is exogenous and the education level of every household is determined forever when it enters the economy.

Employment status. Households in our economy are either workers, disabled households, or retirees. We denote workers by ω, disabled households by d, and retirees by ρ. Consequently, $\mathcal{E} = \{\omega, d, \rho\}$. Every household enters the economy as a worker and every worker faces a positive probability of becoming disabled at the end of each period of their working lives. Once a household has reached

[1]This model economy is an enhancement of the model economy that we presented in Díaz-Giménez and Díaz-Saavedra (2009).

[2]To calibrate our model economy, we use data per person older than 20. Therefore our model economy households are really individual people.

Figure 1: The Endowment of Efficiency Labor Units, the Disability Risk, and the Payroll Tax



A: The Endowment of ELU B: The Disability Risk (%) C: The Payroll Tax*

*In the vertical axis of this panel we plot payroll tax collections expressed as the percentage share of GDP per person over 20 and in the horizontal axis we plot labor income expressed as a multiple of GDP per person over 20.

the first retirement age, which we denote by R_0, it decides whether to retire. Both the disability shock and the retirement decision are irreversible and there is no mandatory retirement age.

Workers. Workers receive an endowment of efficiency labor units every period. This endowment has two components: a deterministic component, which we denote by ϵ_{jh}, and a stochastic component, which we denote by s.

The deterministic component depends on the household age and education, and we use it to characterize the life-cycle profiles of earnings. We model these profiles using the following quadratic functions:[3]

$$\epsilon_{jh} = \xi_{1h} + \xi_{2h} j - \xi_{3h} j^2 \tag{1}$$

We choose this functional form because it allows us to represent the life-cycle profiles of the productivity of workers in a very parsimonious way. We represent the versions of these functions calibrated to Spanish data in Panel A of Figure 1.

The stochastic component is independently and identically distributed across the households, and we use it to generate earnings and wealth inequality within the age cohorts. This process does not depend on either the age or the education of the households, and we assume that it follows a first order, finite state, Markov chain with conditional transition probabilities given by

$$\Gamma[s' \mid s] = \Pr\{s_{t+1} = s' \mid s_t = s\}, \text{ where } s, s' \in S = \{s_1, s_1, \ldots, s_n\}. \tag{2}$$

We assume that the process on s takes three values and, consequently, that $s \in S = \{s_1, s_2, s_3\}$. We make this assumption because it turns our that three states are sufficient to account for the

[3]In the expressions that follow the letters a denote parameters.

Lorenz curves of the Spanish distributions of income and labor earnings in enough detail, and because we want to keep this process as simple as possible.

Disabled households. Each period able-bodied workers of age j and education level h face a probability φ_{jh} of becoming disabled from age $j+1$ onwards. The workers find out whether they have become disabled at the end of the period, once they have made their labor and consumption decisions. When a worker becomes disabled, she exits the labor market and she receives no further endowments of efficiency labor units, but she is entitled to receive a disability pension until she dies.

To choose the values of the probabilities of becoming disabled, we proceed in two stages. First we choose the aggregate probability of becoming disabled. We denote it by q_j, and we assume that it is determined by the following function:

$$q_j = \xi_4 e^{(\xi_5 \times j)} \tag{3}$$

We choose this functional form because the number of disabled people in Spain increases more than proportionally with age, according to the *Boletín de Estadísticas Laborales* (2007).

Once we know the value of q_j, we solve the following system of equations:

$$\begin{cases} q_j \mu_{j,2014} & = & \sum_h \varphi_{jh} \mu_{jh,2014} \\ \varphi_{j2} & = & \xi_6 \varphi_{j1} \\ \varphi_{j3} & = & \xi_7 \varphi_{j1} \end{cases} \tag{4}$$

This procedure allows us to make the disability process dependent on the educational level as is the case in Spain. We represent the values for φ_{jh} calibrated to Spanish data in Panel B of Figure 1.[4]

Retirees. Workers who are R_0 years old or older decide whether to remain in the labor force, or whether to retire and start collecting their retirement pension. They make this decision after they observe their endowment of efficiency labor units for the period. In our model economy retirement pensions are incompatible with labor earnings and, consequently, retirees receive no endowment of efficiency labor units.

Insurance markets. A key feature of our model economy is that there are no insurance markets for the stochastic component of the endowment shock. When insurance markets are allowed to operate, every household of the same age and education level is identical, and the earnings and wealth inequality disappears almost completely.

Assets. Households in our model economy differ in their asset holdings, which are constrained to being positive. Since leisure is an argument of their utility function, this borrowing constraint can

[4]The data on disability can be found at www.empleo.gob.es/es/estadisticas.

be interpreted as a solvency constraint that prevents the households from going bankrupt in every state of the world. These restrictions give the households a precautionary motive to save. They do so accumulating real assets, which we denote by a_t, and which take the form of productive capital. For computational reasons we restrict the asset holdings to belong to the discrete set $\mathcal{A}=\{a_0, a_1, \ldots, a_n\}$. We choose $n=99$, and assume that $a_0=0$, that $a_{99}=75$, and that the spacing between any consecutive two points in set $\mathcal{A}$ is constant.[5]

Pension rights. Workers also differ in the pension rights which they accumulate when they pay payroll taxes. These rights are used to determine the value of their pensions when they retire. The rules of the pension system, which we describe below, include the rules that govern the accumulation of pension rights, and the rules that determine the mapping from pension rights into pensions. In our model economy workers take this mapping into account when they decide how much to work and when to retire. We assume that pension rights belong to the discrete set $B_t=\{b_{0t}, b_{1t}, \ldots, b_{mt}\}$, that $m = 9$, and that the spacing between points in set B_t is increasing.[6] We also assume that $b_{0t} = 0$, and that $b_{mt} = a_{15}\bar{y}_t$, where $a_{15}\bar{y}_t$, is the maximum earnings covered by the Spanish pension system.

Pensions. Disabled households differ in their disability pensions and retirees differ in their retirement pensions. We assume that both the disability and retirement pensions belong to the set $P_t=\{p_{0t}, p_{1t}, \ldots, p_{mt}\}$. Since this mapping is single valued, and the cardinality of the set of pension rights, B_t, is 10, we let $m = 9$ also for P_t. We also assume that the distances between any two consecutive points in P_t are increasing. We make this assumption because minimum pensions play a large role in the Spanish system and this suggests that we should have a tight grid in the low end of P_t.

Preferences. We assume that households derive utility from consumption, $c_{jht} \geq 0$, and from non-market uses of their time, $(1 - l_{jht})$, and that their preferences can be described by the standard Cobb-Douglas expected utility function that we describe in expression (25).

1.2 The Firm

In our model economy there is a representative firm. We assume that aggregate output, Y_t, depends on aggregate capital, K_t, and on the aggregate labor input, L_t, through a constant returns to scale, Cobb-Douglas, aggregate production function of the form

$$Y_t = K_t^{\theta}(A_t L_t)^{1-\theta} \tag{5}$$

[5] In overlapping generation models with finite lives and no altruism there is no need to impose an upper bound for set $\mathcal{A}$ since households who reach the maximum age will optimally consume all their assets. İmrohoroğlu, İmrohoroğlu, and Joines (1995) make a similar point.

[6] Set B_t changes with time because its upper bound is the maximum covered earnings which are proportional to per capita output.

where A_t denotes an exogenous labor-augmenting productivity factor whose law of motion is $A_{t+1} = (1 + \gamma_t) A_t$, and where $A_0 > 0$. Aggregate capital is obtained aggregating the capital stock owned by every household, and the aggregate labor input is obtained aggregating the efficiency labor units supplied by every household. We assume that capital depreciates geometrically at a constant rate, δ, and we use r and w to denote the prices of capital and of the efficiency units of labor before all taxes. We also assume that factor and product markets are perfectly competitive.

1.3 The Government

The government in our model economy taxes capital income, household income and consumption, and it confiscates unintentional bequests. It uses its revenues to consume, and to make transfers to households other than pensions. In addition, the government runs a pay-as-you-go pension system.

The consolidated government and pension system budget constraint is

$$G_t + P_t + Z_t = T_{kt} + T_{st} + T_{yt} + T_{ct} + E_t + [(F_t(1 + r^*) - F_{t+1}] \tag{6}$$

where G_t denotes government consumption, P_t denotes total pension payments, Z_t denotes government transfers other than pensions, T_{kt}, T_{st}, T_{yt}, and T_{ct}, denote the revenues collected by the capital income tax, the payroll tax, the household income tax, and the consumption tax, E_t denotes unintentional bequests, and $F_t > 0$ denotes the value of the pension reserve fund at the beginning of period t, and r^* denotes the exogenous interest rate that the government obtains from the pension reserve fund assets.

Government consumption. We assume that the sequence of government consumption is exogenous.

Pensions. We describe pension expenditures in Section 1.4 below.

Other transfers. We assume that transfers other than pensions are thrown to the sea so that they create no distortions in the household decisions.

Capital income taxes. Capital income taxes are described by the following function

$$\tau_k(y_t^k) = a_1 y_t^k \tag{7}$$

where y_t^k denotes before-tax capital income.

Payroll taxes. We describe payroll taxes in Section 1.4 below.

Household income taxes. Household income taxes are described by the function

$$\tau_y(y_t^b) = a_2 \left\{ y_t^b - \left[a_3 + (y_t^b)^{-a_4} \right])^{-1/a_4} \right\} \tag{8}$$

where the tax base is

$$y_t^b = y_t^k + y_t^l + p^d(b_t) + p(b_t) - \tau_k(y_t^k) - \tau_s(y_t^l) \tag{9}$$

where y_t^l denotes before-tax labor income, and τ_s denotes the payroll tax function that we describe below. Expression (8) is the function chosen by Gouveia and Strauss (1994) to model effective personal income taxes in the United States, and it is also the functional form chosen by Calonge and Conesa (2003) to model effective personal income taxes in Spain.

Consumption taxes. Consumption taxes are described by the function

$$\tau_c(c_t) = a_{5t}c_t. \tag{10}$$

Estate taxes. We assume that the assets that belong to the households that exit the economy are confiscated by the government.

The pension reserve fund. Expression $[F_t(1+r^*) - F_{t+1}]$ denotes the revenues that the government obtains from the pension reserve fund or that deposits into it. We assume that the pension reserve fund is non-negative and we describe its law of motion in Section 1.4 below.

1.4 The Pension System

To complete the specification of our model economy we need to describe its pay-as-you-go pension system. A pay-as-you-go pension system is a payroll tax, the rules that govern the accumulation of pension rights, and the rules that map pension rights into pensions. These rules include the rules that specify the legal retirement ages and the rules that describe the revaluation of pensions. In our benchmark model economy we choose the payroll tax and the pension system rules so that they replicate as closely as possible the Spanish pay-as-you-go pension system in 2014, which is our chosen benchmark model economy calibration year.

Retirement Ages. In Spain in 2014 the retirement age that entitled workers to receive a full retirement pension was 65 for the workers who had contributed during at least 35 years and 6 months. Workers with a shorter contributive period were required to retire at 65 years and two months. Workers aged 61 or older could retire earlier paying an early retirement penalty, as long as they had contributed to the pension system for at least 30 years, and when the decision to retire had not been made by the worker. Workers who decided to retire voluntarily were required to be 63 years and two months old, as long as they had contributed to the pension system for at least 35 years. The 2011 and 2013 Pension Reforms delayed these legal retirement ages gradually. This delay will be completed in 2027 when the normal retirement age will reach 67 and the minimum retirement age will reach 65.[7]

In our model economy the early retirement age is R_0 and the normal retirement age is R_1. In 2014 these ages are 61 and 65 years. We delay the legal retirement ages to 62 and 66 years in 2018, and to 63 and 67 years in 2024. See Díaz-Giménez and Díaz-Saavedra (2016) for the details.

Covered Earnings. The Spanish pension system puts a limit on pensionable earnings. Therefore, in many cases, the earnings covered by the pension system are less than the actual earnings. In our model economy we denote the covered earnings by $\hat{y}^l_{it}$, and we define them as follows:

$$\hat{y}^l_{jt} = \min\{y^l_{jt}, y_{\max,t}\} \tag{11}$$

where $y_{\max}$ denotes the maximum covered earnings. We model maximum covered earnings as a constant proportion, a_6, of per capita output at market prices at every period t. Formally

$$y_{\max,t} = a_6 \bar{y}_t \tag{12}$$

Payroll Taxes. In Spain payroll tax rates are proportional to covered earnings, which are defined as total earnings, excluding payments for overtime work. In 2014 the payroll tax rate was 28.3 percent, of which 23.6 percent was attributed to the employer and the remaining 4.7 percent to the employee; maximum monthly covered earnings were 3,198 euros.[8] In our model economy the payroll tax rate is a_8, and the payroll tax function is the following:

$$\tau_s(y^l_{jt}) = \begin{cases} a_7\bar{y}_t - \left[a_7\bar{y}_t \left(1 + \frac{a_8 y^l_{jt}}{a_7 \bar{y}_t}\right)^{-y^l_{jt}/a_8\bar{y}_t} \right] & \text{if } j < R_1 \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

where a_7 is the cap of the payroll tax, $\bar{y}_t$ is per capita output *at market prices*, y^l_{jt} is labor income, j is the household's age, and R_1 denotes the normal retirement age. Parameter a_8 controls the slope of the tax function, and we choose its value to match the value of the Spanish payroll-tax collections to output ratio.

Early Retirement Penalties. The 2011 and 2013 Pension Reforms established the following early retirement penalties: The early retirement penalties are 7.5 percent per year for households who had contributed between 30 and 34 years; 7 percent per year for households who had contributed between 35 and 37 years; 6.5 percent per year for households who had contributed between 38 and 39 years; and 6 percent per year for households who had contributed for 40 years or more.

[7] The early retirement limit will reach 63 years in 2017 for workers whose retirement decision is not voluntary.

[8] Covered earnings ceilings vary with broadly defined professional categories. In 2014 there were eleven of these categories, but the effective number of caps was only five.

As we describe below, in our model economy we abstract from the durations of the contributory careers, and workers who choose to retire between ages R_0 and R_1 pay an early retirement penalty, λ_j, which is determined by the following function

$$\lambda_j = \begin{cases} a_9 - a_{10}(j - R_0) & \text{if } R_0 \leq j < R_1 \\ 0 & \text{if } j \geq R_1 \end{cases} \tag{14}$$

where a_9 and a_{10} are the parameters which we choose to replicate the Spanish early retirement penalties. Specifically, the annual early-retirement penalty is 7 percent per year.

The Sustainability Factor. The 2011 and 2013 Pension Reforms also introduce a demographic Sustainability Factor (SF) which will be applied from 2019 onwards. This factor adjusts new pensions to the life-expectancy of cohorts aged 67 so that life-time pension wealth is approximately the same for every cohort. Following the Spanish rules, we assume that the law of motion of the SF is the following:

$$SF_t = \varepsilon_t SF_{t-1} \tag{15}$$

where ε_t is a time-varying measure of the relative life-expectancy at age 67. Specifically, for the period 2019–2023 the value of ε will remain constant at

$$\varepsilon_t = \left[\frac{e_{67,2012}}{e_{67,2017}}\right]^{1/5} \tag{16}$$

In this expression variable $e_{67,t}$ denotes the life expectancy at age 67 in year t. For the period 2024–2028 the value of ε will be updated to

$$\varepsilon_t = \left[\frac{e_{67,2017}}{e_{67,2022}}\right]^{1/5} \tag{17}$$

and so on.[9]

Retirement Pensions. In Spain, at least 15 years of contributions are required to be entitled to receive a contributive retirement pension. In general, these pensions are incompatible with labor income. The method used to calculate the pensions is earnings-based. Pension benefits depend on the amounts contributed, on the number of years of contributions, on the retirement age, and on the values of the Sustainability Factor —for first pensions— and of the Pension Revaluation Index —for all other pensions.[10]

In 2014 the *regulatory base* was defined as the average labor earnings of the last 17 years before retirement. This number will increase gradually one year each year until it reaches 25 years in

[9] Before 2019 $\varepsilon_t = 1$.

[10] The pension is 50 percent of the regulatory base when the number of years of contributions is 15, and this percentage increases with the duration of the contributory career.

2022.[11] Taking all this rules into account, in Spain the first pension of a household that retires at age $j \geq R_0$ is calculated according to the following formula:

$$p_{jt} = \phi(N) SF_t (1.03)^v (1 - \lambda_j) \frac{1}{N_b} \sum_{i=j-N_b}^{j-1} \hat{y}^l_{i,t+i-R_0} \tag{18}$$

In this formula, function $\phi(N)$ denotes the pension system's replacement factor which depends on the number of years of contributions, N, in a way that we have described above; parameter v denotes the number of years that the worker remains in the labor force after reaching the normal retirement age; and parameter N_b denotes the number of consecutive years before retirement that are used to compute the retirement pension.

In our benchmark model economy we calculate the retirement pensions using the following formula:

$$p_{jt} = \phi SF_t (1.03)^v (1 - \lambda_j) b_{jt} \tag{19}$$

where b_{jt} denotes the model economy pension rights which we define below. Expression (19) replicates most of the features of Spanish retirement pensions. The main difference is that in our model economy pensions are independent of the number of years of contributions. We abstract from this feature of Spanish pensions for computational reasons.

Pension rights. In our benchmark model economy we calculate pension rights so that they replicate the Spanish pension rights as closely as possible. Formally, in the model economy the expression for the value of the beginning-of-period pension rights is the following:

$$b_{jt} = \begin{cases} \frac{\sum_{i=j-N_b}^{j-1} \hat{y}^l_{i,t+i-R_0}}{N_b} & \text{for} \quad j = R_0, \\ \frac{(N_b - 1) b_{j-1,t-1} + \hat{y}^l_{j-1,t-1}}{N_b} & \text{for} \quad j > R_0, \end{cases} \tag{20}$$

Notice that Expression (20) replicates the Spanish calculation of pension rights exactly for $j = R_0$ and approximately for $j > R_0$. In our model economy, as in Spain, $N_b = 17$ in 2014 and then it increases one year each year until it reaches 25 in 2022.

Minimum and maximum pensions. Spanish pensions are bound by a minimum and a maximum pension. Minimum pensions depend the pensioner's age and on the composition of her household. When an eligible person's pension entitlement is smaller than the minimum pension and she has no other resources, the system tops up her pension entitlement until it reaches the value of the minimum pension. Our model economy introduces this feature. Formally, we require that

$$p_{\min,t} \leq p_t \leq p_{\max,t} \tag{21}$$

[11] Labor income earned in the last two years before retirement entered into the calculation in nominal terms. The labor earnings of the remaining years were revaluated using the rate of change of the Spanish Consumer Price Index.

where $p_{\min,t}$ denotes the minimum pension and $p_{\max,t}$ denotes the maximum pension.

In our benchmark model economy we revaluate all pensions including the minimum and maximum pensions using the Pension Revaluation Index which we describe below.

The Pension Revaluation Index. Until 2013 in Spain minimum and maximum pensions were increased discretionally and all other pensions were revaluated using the Consumer Price Index. Since 2014, all contributive pensions were revaluated according to a *Pension Revaluation Index* (PRI) which is part of the 2013 Pension Reform. The legal definition of the PRI is the following:

$$g_{t+1} = \overline{g}_{c,t+1} - \overline{g}_{p,t+1} - \overline{g}_{s,t+1} + \alpha(\frac{\tilde{R}_{t+1} - \tilde{E}_{t+1}}{\tilde{E}_{t+1}}) \tag{22}$$

where $\overline{x}_t$ denotes the moving arithmetic average of variable x_t computed between $t-5$ and $t+5$, $\tilde{x}$ denotes the moving geometric average of variable x_t computed between $t-5$ and $t+5$, $g_{c,t+1}$ is the growth rate of the pension system revenues, $g_{p,t+1}$ is the growth rate of the number of pensions, $g_{s,t+1}$ is the growth rate of the average pension due to the substitution of old pensions by new pensions, $0.25 \leq \alpha \leq 0.33$ is an adjustment coefficient, R_{t+1} denotes the pension system revenues, and E_{t+1} denotes pension system expenditures.

Finally, the Spanish law specifies two bounds for the PRI. The lower bound is 0.25 percent and the upper bound is 0.5 percent plus the inflation rate. In our model economy we replicate the formula used to calculate the Spanish PRI exactly and we choose an inflation scenario to replicate its bounds.

Disability Pensions. We model disability pensions explicitly for two reasons. First, because disability pensions represent a large share of all Spanish pensions. In 2014, 10.0 percent of all contributive pensions and 14.9 percent of the sum of the retirement and disability pensions paid by the *Régimen General* were disability pensions. Second, because Spaniards often use disability pensions as an alternative route to early retirement. See Boldrin and Jiménez-Martín (2007) for an elaboration of this argument.

The rules used to define pensionable income for workers who qualify for a disability pension in Spain are complex and they depend on detailed individual circumstances and on the type of disability. In our model economy we approximate these rules assuming that the disability pension is 75 percent of the pension rights of the disabled worker and that this amount is bounded below by the minimum retirement pension. Formally, we compute the disability pensions as follows:

$$p_t^d(b_t) = \max\{0.75b_t, p_{\min,t}\}. \tag{23}$$

The Pension Reserve Fund. Since the year 2000, Spain has had a pension reserve fund which is invested in fixed income assets and which is financed with part of the pension system surpluses.

From 2010 onwards, the reserve fund assets have been used to finance the pension system deficits when needed. In 2014, the total amount of assets accumulated in the pension reserve fund was 41,634.23 million euros which corresponded to 4.00 percent of that year's GDP.

In our benchmark model economy, we assume that all the pension system surpluses are deposited into a pension reserve fund which evolves according to

$$F_{t+1} = (1 + r^*)F_t + T_{st} - P_t \tag{24}$$

We require the pension reserve fund to be non-negative. We assume that the pension fund assets are used to finance the pension system deficits. Once the pension reserve fund runs out, we assume that the government changes the consumption tax rate as needed to finance the pensions.

1.5 The Households' Decision Problem

We assume that the households in our model economy solve the following decision problem:

$$\max E \left\{ \sum_{j=20}^{100} \beta^{j-20}\, \psi_{jt}\, (1 - \varphi_{jh})\, [c_{jht}^{\alpha}(1 - l_{jht})^{(1-\alpha)}]^{(1-\sigma)}/1 - \sigma \right\} \tag{25}$$

subject to

$$c_{jht} + a_{jht+1} + \tau_{jht} = y_{jht} + a_{jht} \tag{26}$$

where

$$\tau_{jht} = \tau_k y_{jht}^k + \tau_y(y_{jht}^b) + \tau_{st}(y_{jht}^l) + \tau_{ct} c_{jht} \tag{27}$$

$$y_{jht} = y_{jht}^k + y_{jht}^l + p_t^d(b_t) + p_t(b_t) \tag{28}$$

$$y_{jht}^k = a_{jht} r_t \tag{29}$$

$$y_{jht}^l = \epsilon_{jh} s_t l_{jht} w_t \tag{30}$$

$$a_{jht} \in \mathcal{A}, p_t(b_t) \text{ and } p_t^d(b_t) \in P_t, s_t \in S \text{ for all } t, \text{ and } a_{jh0} \text{ is given,} \tag{31}$$

and where parameter $\beta > 0$ denotes the time-discount factor, function τ_y is defined in expression (8), variable y_{jht}^b is defined in expression (9), function τ_s is defined in expression (13), function p is defined in expression (19), the law of motion of b_t is defined in expression (20), and function p^d is defined in expression (23).

Notice that every household can earn capital income, that only workers can earn labor income, that only disabled households receive disability pensions, and that only retirees receive retirement pensions. As we have already mentioned, an important feature of the households decision problem that we have omitted here is that households decide optimally when to retire, once they have reached age R_0. This decision depends on their state variables, j, h, a_t, s_t, and b_t, and on the expected benefits and costs of continuing to work. The benefits are the labor earnings and, possibly, the reduction of the early retirement penalties or the late retirement premium, and the costs are the forgone leisure and the forgone pension. They also take into account the change in their pension rights, $b_{t+1} - b_t$, which could be a benefit or a cost depending on the values of b_t and of the current and expected future endowments of efficiency labor units.

1.6 Definition of Equilibrium

Let $j \in J$, $h \in H$, $e \in \mathcal{E}$, $a \in \mathcal{A}$, $b_t \in B_t$, and $p_t \in P_t$, and let $\mu_{j,h,e,a,b,p,t}$ be a probability measure defined on $\Re = J \times H \times \mathcal{E} \times \mathcal{A} \times B_t \times P_t$.[12] Then, given initial conditions μ_0, A_0, E_0, F_0, and K_0, a competitive equilibrium for this economy is a government policy, $\{G_t, P_t, Z_t, T_{kt}, T_{st}, T_{yt}, T_{ct}, E_{t+1}, F_{t+1}\}_{t=0}^{\infty}$, a household policy, $\{c_t(j,h,e,a,b,p),\ l_t(j,h,e,a,b,p),\ a_{t+1}(j,h,e,a,b,p)\}_{t=0}^{\infty}$, a sequence of measures, $\{\mu_t\}_{t=0}^{\infty}$, a sequence of factor prices, $\{r_t, w_t\}_{t=0}^{\infty}$, a sequence of macroeconomic aggregates, $\{C_t, I_t, Y_t, K_{t+1}, L_t\}_{t=0}^{\infty}$, a function, Q, and a number, r^*, such that:

(i) The government policy and r^* satisfy the consolidated government and pension system budget constraint described in Expression (6) and the the law of motion of the pension system fund described in Expression (24).

(ii) Firms behave as competitive maximizers. That is, their decisions imply that factor prices are factor marginal productivities $r_t = f_1(K_t, A_t L_t) - \delta$ and $w_t = f_2(K_t, A_t L_t)$.

(iii) Given the initial conditions, the government policy, and factor prices, the household policy solves the households' decision problem defined in Expressions (25), through (31).

(iv) The stock of capital, consumption, the aggregate labor input, pension payments, tax revenues, and accidental bequests are obtained aggregating over the model economy households as follows:

$$K_t = \int a_{jht} d\mu_t \tag{32}$$

$$C_t = \int c_{jht} d\mu_t \tag{33}$$

$$L_t = \int \epsilon_{jh} s_t l_{jht} d\mu_t \tag{34}$$

$$P_t = \int p_t d\mu_t \tag{35}$$

$$T_{ct} = \int \tau_{ct}(c_{jht}) d\mu_t \tag{36}$$

(37)

[12] Recall that, for convenience, whenever we integrate the measure of households over some dimension, we drop the corresponding subscript.

$$T_{kt} = \int \tau_k(y^k_{jht}) d\mu_t \quad (38)$$

$$T_{st} = \int \tau_s(y^l_{jht}) d\mu_t \quad (39)$$

$$T_{yt} = \int \tau_y(y^b_{jht}) d\mu_t \quad (40)$$

$$E_t = \int (1-\psi_{jt}) a_{jht+1} d\mu_t \quad (41)$$

where $y^k_{jht} = a_{jht} r_t$, $y^l_{jht} = \epsilon_{jh} s_t l_{jht} w_t$, and $y^b_{jht} = y^k_{jht} + y^l_{jht} + p_t - \tau_k(y^k_t) - \tau_s(y^l_t)$, and all the integrals are defined over the state space $\Re$.

(v) Net investment I_t is

$$I_t = K_{t+1} - (1-\delta)K_t \quad (42)$$

Notice that in this model economy $I_t \neq \int (a_{jht+1} - (1-\delta)a_{jht}) d\mu_t$. This is because some households die and their end of period capital is confiscated and because immigrants who are older than 20 bring into the economy the same amount of capital as the current residents who are in the same state.

(vi) The goods market clears:

$$C_t + \int (a_{jht+1} - (1-\delta)a_{jht}) d\mu_t + G_t + [Z_t + (F_{t+1} - F_t)] = F(K_t, A_t L_t). \quad (43)$$

The last term of the left-hand side of this expression is not standard. Transfers other than pensions, Z_t, show up in this expression because we assume that the government throws them into the sea. And the change in the value of the pension reserve fund, $(F_{t+1} - F_t)$ shows up because pension system surpluses are invested in the pension fund and pension system deficits are financed with the fund until it is depleted.[13]

(vii) The law of motion for μ_t is:

$$\mu_{t+1} = \int_{\Re} Q_t d\mu_t. \quad (44)$$

Describing function Q formally is complicated because it specifies the transitions of the measure of households along its six dimensions: age, education level, employment status, assets holdings, pension rights, and pensions. An informal description of this function is the following:

[13] The last term of the left-hand side of Expression (43) would show up as net exports in the standard national income and product accounts.

We assume that new-entrants, who are 20 years old, enter the economy as able-bodied workers, that they draw the stochastic component of their endowment of efficiency labor units from its invariant distribution, and that they own zero assets and zero pension rights. Their educational shares are exogenous and they determine the evolution of μ_{ht}. We also assume that new-entrants who are older than 20 replicate the age, education, employment status, wealth, pension rights, and pensions share distribution of the existing population.

The evolution of μ_{jht} is exogenous, it replicates the Spanish demographic projections, and we compute it following a procedure that we describe in Section 2.6 below. The evolution of μ_{et} is governed by the conditional transition probability matrix of its stochastic component, by the probability of becoming disabled, and by the optimal decision to retire. The evolution of μ_{at} is determined by the optimal savings decision and by the changes in the population. The evolution of μ_{bt} is determined by the rules of the Spanish public pension system which we have described in Section 1.1.

2 Calibration

To calibrate our model economy we do the following: First, we choose a calibration target country — Spain in this article— and a calibration target year —2014 in this article. Then we choose the initial conditions and the parameter values that allow our model economy to replicate as closely as possible selected macroeconomic aggregates and ratios, distributional statistics, and the institutional details of our chosen country in our target year. We describe these steps in the subsections below.

2.1 Initial conditions

To determine the initial conditions, first we choose an initial distribution of households, μ_0. In Section 2.6 we provide a detailed description about how we obtain that distribution. The initial distribution of households implies an initial value for the capital stock. This value is $K_{2014} = 12.4435$. The initial distribution of households and the initial survival probabilities determine the initial value of unintentional bequests, E_{2014}. We must also specify the initial values for the productivity process, A_{2014}, and for the pension reserve fund F_{2014}. Since A_{2014} determines the units which we use to measure output and does nothing else, we choose $A_{2014} = 1.0$. Finally, our choice for the initial value of the pension reserve fund is $F_{2014} = 0.04\ Y^*_{2014}$, where Y^*_t denotes output at market prices, which we define as $Y^*_t = Y_t + T_{ct}$. Our choice for F_{2014} replicates the value of the Spanish pension reserve fund at the end of 2014.

2.2 Parameters

Once the initial conditions are specified, to characterize our model economy fully, we must choose the values of a total of 50 parameters. Of these 50 parameters, 3 describe the household preferences, 21 the process on the endowment of efficiency labor units, 4 the disability risk, 3 the production technology, 12 the pension system rules, and 7 the remaining components of the government policy. To choose the values of these 50 parameters we need 50 equations or calibration targets which we describe below.

2.3 Equations

To determine the values of the 50 parameters that identify our model economy, we do the following. First, we determine the values of a group of 31 parameters directly using equations that involve either one parameter only, or one parameter and our guesses for (K, L). To determine the values of the remaining 19 parameters we construct a system of 19 non-linear equations. Most of these equations require that various statistics in our model economy replicate the values of the corresponding Spanish statistics in 2014. We describe the determination of both sets of parameters in the subsections below.

2.3.1 Parameters determined solving single equations

The life-cycle profile of earnings. We measure the deterministic component of the process on the endowment of efficiency labor units independently of the rest of the model. We estimate the values of the parameters of the three quadratic functions that we describe in Expression (1), using the age and educational distributions of hourly wages reported by the *Instituto Nacional de Estadística* (INE) in the *Encuesta de Estructura Salarial* (2010) for Spain.[14] This procedure allows us to identify the values of 9 parameters directly.

The disability risk. We want the probability of becoming disabled to approximate the data reported by the *Boletín de Estadísticas Laborales* (2007) for the Spanish economy. We use this dataset to estimate the values of parameters ξ_4 and ξ_5 of Expression (3) using an ordinary least squares regression of q_j on j. According to the *Instituto de Mayores y Servicios Sociales*, in 2008 in Spain 62.6 percent of the total number of disabled people aged 25 to 44 years old had not completed high school, 26.9 percent had completed high school, and the remaining 10.5 percent had completed college. We use these shares to determine the values of parameters ξ_6 and ξ_7 of Expression (4). Specifically, we choose $\xi_6 = 0.269/0.626 = 0.4297$ and $\xi_7 = 0.105/0.626 = 0.1677$. This procedure allows us determine the values of 4 parameters directly.

[14]Since we only have data until age 64, we estimate the quadratic functions for workers in the 20–64 age cohort and we project the resulting functions from age 65 onwards.

Table 1: Parameters determined solving single equations

	Parameter	Value
Parameters determined directly		
Earnings Life-Cycle		
	$\xi_{1,1}$	0.9189
	$\xi_{1,2}$	0.8826
	$\xi_{1,3}$	0.5064
	$\xi_{2,1}$	0.0419
	$\xi_{2,2}$	0.0674
	$\xi_{2,3}$	0.1648
	$\xi_{3,1}$	0.0006
	$\xi_{3,2}$	0.0008
	$\xi_{3,3}$	0.0021
Disability Risk		
	ξ_4	0.000449
	ξ_5	0.0924
	ξ_6	0.4297
	ξ_7	0.1677
Preferences		
Curvature	σ	4.0000
Technology		
Capital share	θ	0.3669
Productivity growth rate	γ	0.0000
Public Pension System		
Maximum early retirement penalty	a_9	0.2800
Early retirement penalty per year	a_{10}	0.0700
Number of years of contributions	N_b	17
First retirement age	R_0	61
Normal retirement age	R_1	65
Rate of return for the pension fund	r^*	0.0200
Government Policy		
Household Income Tax function		
	a_9	0.4500
	a_{11}	1.0710
Parameters determined by guesses for (K, L)		
Public Pension System		
Payroll tax cap	a_7	0.5096
Maximum covered earnings	a_6	1.8000
Minimum retirement pension	b_{0t}	0.6877
Maximum retirement pension	b_{mt}	4.8013
Government Policy		
Government consumption	G	0.7496
Capital income tax rate	a_1	0.2129
Consumption tax rate	a_5	0.2249

The pension system. In 2014 in Spain, the payroll tax rate paid by households was 28.3 percent and it was levied only on the first 50,358 euros of annual gross labor income. Hence, the maximum contribution was 14,251 euros which correspond to 50,96 percent of the Spanish GDP per person who was 20 or older. To replicate this feature of the Spanish pension system we choose the value of parameter a_7 of our payroll tax function to be $a_7 = 0.5096$.

Our choice for the number of years used to compute the retirement pensions in our benchmark model economy is $N_b = 17$. This is because in 2014 the Spanish *Régimen General de la Seguridad Social* took into account the last 17 years of contributions prior to retirement to compute the pension.

We assume that the minimum pension, the maximum pension, and the maximum covered earnings are directly proportional to per capita income. Our targets for the proportionality coefficients are $b_{0t} = 0.18321\overline{y}_t$, $b_{mt} = 1.2790\overline{y}_t$, and $a_6 = 1.8$. These numbers correspond to their values in 2014 .[15]

In the benchmark model economy we choose the first and the normal retirement ages to be $R_0 = 61$ and $R_1 = 65$. To identify the early retirement penalty function, we choose $a_9 = 0.28$, and $a_{10} = 0.07$. This is because we have chosen $R_0 = 61$. Finally, for the rate of return on the pension reserve fund's assets we choose $r^* = 0.02$.[16] These choices allow us to determine the values of 10 parameters.

Government policy. To specify the government policy, we must choose the values of government consumption, G_t, of the tax rate on capital income, a_1, of parameters a_2 and a_3 of the household income tax function, and of the tax rate on consumption, a_{5t}. We describe our procedure to choose the value of these 5 parameters in Section 2.5.

Preferences. Of the four parameters in the utility function, we choose the value of only σ directly. Specifically, we choose $\sigma = 4.0$. This choice and the value of the share of consumption in the utility function, imply that the relative risk aversion in consumption is 1.8937, which falls within the 1.5-3 range which is standard in the literature.

Technology. According to the OECD data, the capital income share in Spanish GDP was 0.3669 in 2008. Consequently, we choose $\theta = 0.3669$. We also choose the growth rate of total factor productivity directly. We discuss this choices for the growth rate scenarios in the main body of the paper.

[15]Specifically, in 2014 the minimum retirement pension in Spain was 5,122 euros, the maximum pension was 35,762 euros, the maximum covered earnings were 50,358 euros, and GDP per person who was 20 or older was 27,960 euros. All these data are yearly.

[16]In Díaz-Giménez and Díaz-Saavedra (2009) we also run simulations for $r^* = 1$, 3, and 4 percent. We found that the changes implied by the various values of r^* were small and that they did not modify the qualitative conclusions of that article.

Adding up. So far we have determined the values of 31 parameters either directly or as functions of our guesses for (K, L) only. We report their values in the first two blocks of Table 1.

2.3.2 Parameters determined solving a system of equations

We still have to determine the values of 19 parameters. To find the values of those 19 parameters we need 19 equations. Of those equations, 14 require that model economy statistics replicate the value of the corresponding statistics for the Spanish economy in 2014, 4 are normalization conditions, and the last one is the government budget constraint that allows us to determine the value of Z/Y^* residually.

Table 2: Macroeconomic Aggregates and Ratios in 2014 (%)

	C/Y^{*a}	T_y/Y^*	T_s/Y^*	P/Y^*	K/Y^{*b}	l^c
Spain	56.4	7.7	9.6	10.6	3.28	37.5

[a]Variable Y^* denotes GDP at market prices.
[b]The target for K/Y^* is in model units and not in percentage terms, and it is the Spanish capital to output ratioin 2010.
[c]Variable l denotes the average share of disposable time allocated to the market.

Aggregate Targets. According to the Spanish *Encuesta de Empleo del Tiempo (2010)*, the average number of hours worked per worker was 36.79 per week. If we consider the endowment of disposable time to be 14 hours per day, the total amount of disposable time is 98 hours per week. Dividing 36.79 by 98 we obtain 37.5 percent which is the share of disposable time allocated to working in the market that we target. Consequently, the Frisch elasticity of labour supply implied in our model economy is 0.77, which is in the middle of the range of recent econometric estimates of this parameter.[17]

According to the BBVA database, in 2010 the value of the Spanish capital stock was 3,454,401 million 2000 euros.[18] According to the *Instituto Nacional de Estadística* in 2010 the Spanish Gross Domestic Product at market prices was 1,051,342 million 2000 euros. Dividing these two numbers, we obtain $K/Y = 3.28$, which is our target value for the model economy capital to output ratio. In Section 2.4 we describe how we obtain the values of the first four macroeconomic ratios that we report in Table 2 .

Distributional Targets. We target the 3 Gini indexes and 5 points of the Lorenz curves of the Spanish distributions of earnings, income and wealth for 2004. We have taken these statistics from Budría and Díaz-Giménez (2006), and we report them in bold face in Table 9. Castañeda et al.

[17]See, for example, Fuster et al. (2007).
[18]This number can be found at $http://www.fbbva.es/TLFU/microsites/stock09/fbbva\_stock08\_index.html$.

(2003) argue in favor of this calibration procedure to replicate the inequality reported in the data. These targets give us a total of 8 additional equations.

Normalization conditions. In our model economy there are 4 normalization conditions. The transition probability matrix on the stochastic component of the endowment of efficiency labor units process is a Markov matrix and therefore its rows must add up to one. This gives us three normalization conditions. We also normalize the first realization of this process to be $s(1)=1$.

The Government Budget. Finally, the government budget is an additional equation that allows us to obtain residually the government transfers to output ratio, Z_t/Y_t^*.

2.4 The expenditure ratios

The Spanish National Income and Product Data reported by the *Instituto Nacional de Estadística* (INE) for 2014 are the following:

Table 3: Spanish GDP and its Components for 2014 at Current Market Prices

	Millon Euros	Shares of GDP (%)
Private Consumption	595,791	57.45
Public Consumption	202,437	19.51
Consumption of Non-Profits	11,037	1.06
Gross Capital Formation	204,107	19.67
Exports	338,769	32.67
Imports	313,668	30.24
Total (GDP)	1,037,250	100.00

We adjust the amounts reported in Table 3 according to Cooley and Prescott (1995) and we obtain the following numbers:

– Adjusted Private Consumption: Private Consumption – Private Consumption of Durables + Consumption of Non-Profits = $595,791 - 35,391 + 11,037 = 571,437$ million euros.

– Adjusted Public Consumption: Public Consumption = 202,437 million euros.

– Adjusted Investment (Private and Public): Gross Capital Formation + Private Consumption of Durables = $204,107 + 35,391 = 239,498$ million euros.

The next adjustment is to allocate Net Exports to our measures of C, I, and G. To that purpose, we compute the shares of each of those three variables in the sum of the three and we allocate Net Exports according to those shares. The sum of the three variables is 1,013,372 million euros and the shares of C, I, and G are 56.38, 23.63, and 19.97 percent.

Net Exports are 25,973 million euros. When we allocate them to C, I, and G we obtain the final adjusted values for C, I, and G which are 586,080, 245,635, and 207,623. Naturally, this new adjusted values now add to Total GDP but the adjusted shares remain unchanged and they are 56.38, 23.63, y 19.97 percent of GDP.

Next we redefine the model economy's output and consumption from factor cost to market prices as follows: $Y^* = Y + T_c$, where Y^* is the model economy's output at market prices and T_c is the consumption tax collections, and $C^* = C + T_c$, where C^* is the model economy's consumption at market prices. Finally we use $C^*/Y^* = 56.38$ and $G/Y^* = 19.97$ as targets.

2.5 The government policy ratios

In Table 4 we report the 2014 revenue and expenditure items of the consolidated Spanish public sector. Notice that the GDP share of Government consumption differs from the one that we have computed in Section A3.1 because here we use its unadjusted value.

Table 4: Spanish Public Sector Expenditures and Revenues in 2014*

Expenditures	Millions of euros	Percentage of GDP	Revenues	Millions of euros	Percentage of GDP
Consumption	202,437	19.44	Sales and gross receipts taxes[a]	99,226	9.53
Investment	21,834	2.09	Payroll taxes	100,441	9.64
Pensions	110,208	10.58	Individual income taxes	80,589	7.74
Interest payments	35,490	3.40	Corporate profit taxes	20,891	2.00
Other	93,287	8.96	Other revenues	100,784	9,68
			Deficit	61,319	5.88
Total	463,041	44.47	Total	463,041	44.47

Source: Spanish National Institute of Statistics, Spanish Social Security, and Eurostat.
*Shares of nominal GDP at market prices.
[a]It includes the tax collections from the Value Added Tax and other taxes on products.

If we ignore the public pension system, the government budget in the model economy in 2014 can be written as

$$G_{2014} + Z_{2014} = T_{c,2014} + T_{k,2014} + T_{y,2014} + E_{2014} \tag{45}$$

We target the output shares of $T_{c,2014}$, $T_{k,2014}$, and $T_{y,2014}$ so that they replicate the GDP shares of Sales and Gross Receipt Taxes, Corportate Profit Taxes, and Individual Income taxes. We have already targeted the output ratio of government consumption, G_{2014}, and we have already accounted for government investment. Unitentional bequests, E_{2014}, are exogenous. We define the output share of transfers other than pensions, Z_{2014}, residually to satisfy the budget. We report the model economy government budget items in Table 5 below.

Table 5: Model Economy Public Sector Expenditures and Revenues in 2014 (%Y^*Shares)

Expenditures		Revenues	
Consumption and Investment (G)	19.97	Consumption taxes (T_c)	9.53
Pensions (P)	10.65	Payroll taxes (T_s)	9.62
Other Transfers (Z)	1.33	Household income taxes (T_y)	7.75
		Capital Income Taxes (T_k)	2.00
		Unitentional Bequests (E)	3.05
Total	31.95	Total	31.95

2.6 The initial distribution of households

The initial distribution of households. Recall that $\mu_{j,h,e,a,b,p,t}$ denotes the measure of households of type (j, h, e, a, b, p) at period t and that, whenever we integrate the measure of households over some dimension, we drop the corresponding subscript. To obtain μ_{2014}, we proceed as follows:

1. We take the measure $\mu_{j,2014}$ for all $j = \{20, 21, ..., 100\}$ directly from the 2012 demographic projection for the Spanish economy published by the *National Institute of Statistics* (INE). These demographic projections take into account the forecasts for the net migratory flows into Spain. However, to solve the households' decision problem we use the survival probabilities only.[19]

2. We obtain $\mu_{j,h,2014}$ directly from the *Encuesta de Población Activa*, which reports the educational distribution of the working age population for various age groups.

3. Next, we solve the decision problem of the model economy households. We obtain $\mu_{20,h,e,2014}$ from $\mu_{20,h,2014}$ and the invariant distribution of the stochastic component of the endowment of efficiency labor units process.[20]

   To compute $\mu_{j,h,e,2014}$ for $j = \{21, 22, ..., 100\}$, we use the conditional transition probability matrix of the stochastic component of the endowment of efficiency labor units process, the probability of becoming disabled, and the optimal decision to retire.

4. To obtain $\mu_{20,h,e,a,b,2014}$, we assume that new-entrants own zero assets and have zero pension claims. For $j = 21, 22, ..., 44$, we use the household's optimal saving decisions at age $j - 1$

[19] The survival probabilities can be found at http://www.ine.es/jaxi/menu.do?type=pcaxis&path=%2Ft20%2Fp25t1&file=inebase&L=0.

[20] Note that we have assumed that there are no disabled households of age 20.

and the pension system rules. From age $R_0 - N_b$ onwards, we average the labor income to determine the pension claims and the optimal labor supply decisions.

5. Finally, to obtain $\mu_{j,h,e,a,b,p,2014}$, we use the optimal retirement decisions and the pension system rules.

   Notice that steps 3, 4 and 5 must be computed simultaneously in the same loop.

2.7 The demographic transition

We use the 2012 demographic projections of the *Instituto Nacional de Estadística.* The INE reports and projects the age distribution of Spanish residents from 2012 to 2052 for people aged from zero to 100 and more. Call those age cohorts N_{jt} and let $N_t = \sum_{j=20}^{100+} N_{jt}$. Then, the age distribution of the households in our model economy is $\mu_{jt} = N_{jt}/N_t$ for $j = 20, 21, ..., 99, 100+$ and for $t = 2014, 2015, ..., 2052$.

To extrapolate the distribution of households to 2100, we do the following

- We have assumed that Model Economy ESP and our four Catalonian Model Economies share the age-dependent, time variant, survival probabilities estimated by the INE in its 2012 demographic projection for the period 2014-2051.

- Between 2014 and 2051 the growth rates of these survival probabilities are positive. But these growth rates decrease in an *almost* linear fashion during that period. In fact, the trend is slightly convex.

- To propagate the survival probabilities between 2052 and 2100, we assume that the growth rates decrease *linearly* between those years. Specifically, if $g_{j,t}$ is the growth rate of the survival probability from age j to age $j+1$ at period t, our assumption of linearity implies that, from 2052 onwards, $g_{j,t} = g_{j,t-1} + [g_{j,t-1} - g_{j,t-2}]$ with $g_{j,t-1} < g_{j,t-2}$, so we can compute the entire sequence of growth rates for each age and period between 2052 and 2100, once we have computed the sequence of growth rates between 2014 and 2051. If $g_{j,t}$ becomes negative, we assume that this growth rate is 0.

- Once we have computed the survival probability growth rates, we can compute the evolution of the age-dependent survival probabilities. Specifically, we compute $\psi_{j,t} = \psi_{j,t-1}(1 + g_{j,t})$ for the period 2052 to 2100. From 2100 onwards, we assume that these survival probabilities are constant.

- The next step is to compute the population growth rates, n_t. The population growth rates of Spain and Catalonia are almost constant towards the end of the demographic projection period. Specifically, in 2051 the population growth rates are –0.35 and 0.09 percent. To extend the demographic projection, we assume that the population growth rates remain constant at those values between 2052 and 2150.

- Finally, to project the population sequences between 2052 and 2150, we do the following:

$$N_{j,t} = \begin{cases} \psi_{j-1,t-1} N_{j-1,t-1} & \text{if } j > 20 \\ (1+n_t) \sum_{i=1}^{J} N_{i,t-1} - \sum_{i=2}^{J} N_{i,t} & \text{if } j = 20 \end{cases} \tag{46}$$

where $N_{j,t}$ is the number of persons aged j at period t.

2.8 The educational transition

To update the distribution of education, we assume that from 2015 onwards, 7.33 percent of the 20 year-old entrants have not completed their secondary education, that 62.62 percent have completed their secondary education, and that 30.05 percent have completed college. This was the educational distribution of Spanish households born between 1980 and 1984, which was the most educated cohort in 2014.[21] We also assume that immigrants have the same educational distribution as the residents of the same age.

2.9 Results

In this section we show that our calibrated, benchmark model economy replicates reasonably well most of the Spanish statistics that we target in our calibration procedure.

Table 6: The Stochastic Component of the Endowment Process

		Transition Probabilities			
	Values	$s' = s_1$	$s' = s_2$	$s' = s_3$	$\pi^*(s)^a$
$s = s_1$	1.0000	0.9417	0.0582	0.0000	31.41
$s = s_2$	2.0856	0.0319	0.9680	0.0000	57.25
$s = s_3$	11.2892	0.0000	0.0002	0.9997	11.32

[a] $\pi^*(s)\%$ denotes the invariant distribution of s.

The stochastic component of the endowment process. The procedure that we have used to calibrate our model economy identifies the stochastic component of the endowment of efficiency labor units process, s. In Table 6 we report its main features. Recall that we have restricted to three the number of realizations of s. We find that the value of the highest realization of s is 11.3 times that of its lowest value. We find also that the process on s is very persistent. Specifically, the expected durations of the shocks are 17.2, 31.3, and an astonishing 3333.3 years. In the last column of

[21] This is a similar approach used in Conde-Ruiz and González (2013).

Table 6 we report the invariant distributions of the shocks. We find that approximately 89 percent of the workers are either in state $s = s_1$ or in state $s = s_2$, and that about 11 percent are in state $s = s_3$.[22] These features allow us to replicate reasonably well the Lorenz curves of the Spanish income and earnings distributions, as we report below.

Retirement behavior. In Table 7 we report the average retirement ages and the participation rates of people aged from 60 to 64. The average retirement age in our model economy is 63.7 years, 1.5 years higher than in Spain. We also find that the average retirement ages are increasing in the number of years of education. Unfortunately, we could not find these data for Spain, but we think that this increasing relationship is very plausible, since the Spanish participation rates of the 60–64 age cohort are strongly increasing in education (see the third column of Table 7).

Table 7: Retirement Ages and Participation Rates of Older Workers

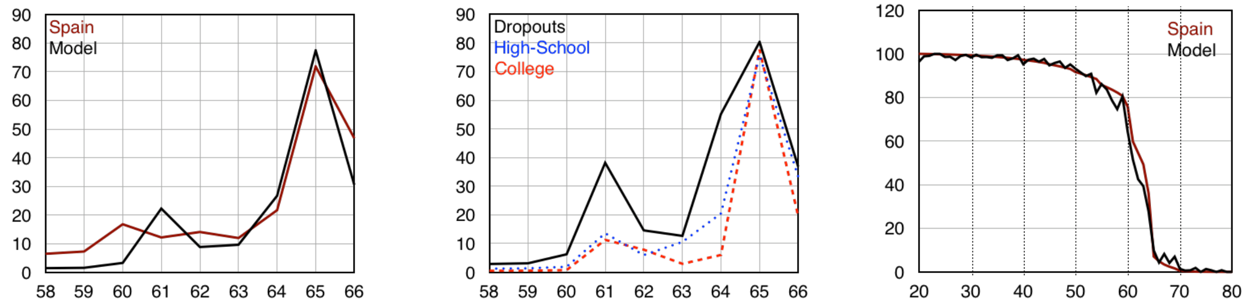
	Avg Ret Ages		Part rates at 60-64 (%)	
	Spain[a]	Model	Spain[b]	Model
All	62.2	63.7	56.6	55.4
Dropouts	*n.a.*	63.0	45.5	36.7
High School	*n.a.*	64.1	61.0	66.8
College	*n.a.*	64.5	75.2	79.3

[a]The Spanish data is for males in 2014 (Source: Eurostat).
[b]The Spanish data is from both the *Encuesta de la Población Activa*, and the *Encuesta de Empleo del Tiempo 2010*, excluding the unemployed and non-participants who do not collect either retirement or disability pensions.

The total participation rate of the households in the 60 to 64 age cohort is 55.4 percent in our model economy. In Spain this number is 56.6 percent. As we have already mentioned, the participation rates both in Spain and in our model economy are increasing in education. This is mainly because, even though all educational types value leisure equally in our model economy, the foregone labor income—which is the opportunity cost of leisure—is lower for less educated workers and, therefore they tend to retire earlier. Our model economy replicates this behavior even though it has fewer labor market categories than Spain. In Spain people of working age can be employed, unemployed, retired, disabled, and other non-participants. In our model economy, we abstract from the unemployed and from the other non-participants. In spite of these differences, we find that the participation rates of older-workers in Spain and in our model economy are similar. We also find that dropouts retire earlier in our model economy than in Spain, and that more educated workers retire work later.

[22]The process on s is very different from the one we found in Díaz-Giménez and Díaz-Saavedra (2009). Specifically, we find that the range of the values of the realizations is larger and that the shocks are more persistent. These differences are mostly due to the progressivity of the personal income tax, the double taxation of capital income, the increase in the share of college educated workers, the change in distributional targets that occur because we delay the calibration year, and the assumption that transfers other than pensions are thrown into the sea.

Figure 2: Retirement Hazards and Shares of Workers (%)*



A: Retirement Hazards (All) B: Retirement Hazards (Edu) C: Shares of Workers (All)

*The Spanish data for the retirement hazards is taken from García Pérez and Sánchez-Martín (2010). The shares of workers are the shares of workers in the sum of workers, disabled people, and retirees. We compute this share for Spain from the *Encuesta de Empleo del Tiempo (2010)*, reported by the INE.

In Panel A of Figure 2 we illustrate the age-profiles of the retirement hazards. The Spanish profile, which displays a small peak at 60 which is the early retirement age and a much larger one at 65 which is the normal retirement age, is a common stylized fact in countries that run defined-benefit pension systems (see Gruber and Wise, 1999). At first sight, our model economy replicates this pattern, although the first peak is at age 61, since this is the first retirement age in the model. A closer scrutiny reveals that the hazard is ten percentage points higher in our model economy at age 65.

In Panel B of Figure 2 we show that high-school dropouts have a higher probability to exit the labor force at age 61 than more educated workers. Our results show that in our model economy 64 percent of those who retire at 61 are dropouts. This finding is consistent with those of Sánchez-Martín (2010), who reports that low-income workers have a higher probability of retiring at age 60 than high-income workers.

The details of the Spanish minimum retirement pension are one of the reasons behind this result. In 2010, about 27 percent of the Spanish retirees receive the minimum retirement pension—this share is 33 percent in our model economy in 2014. Workers who qualify for the minimum pension can start to collect it at the first retirement age without paying an early retirement penalty. Moreover, for many of these workers, remaining in the labor force after this age does not increase their pensions. Since many of these typically low-wage earners gain very little from continued employment, many of them choose to retire as early as possible. In our model economy, 80 percent of the workers who retire at the first retirement age collect the minimum pension, while Jiménez-Martín and Sánchez-Martín (2006) report that in 1997 this number was 67 percent in Spain.

Retirement hazards are lower after the first retirement age, both in Spain and in our model economy. This is because workers who qualify to collect a pension that is higher than the minimum pension and who choose to work for one extra year after this age reduce the early retirement penalty by 7 percent. This means that these workers face an implicit subsidy if they continue to work between ages 61 and 64, and this subsidy may amount to as much as 25 percent of their net yearly salary, as shown by Boldrin et al. (1997)[23].

This behavior changes at age 65. This is because the incentives provided by the Spanish pension system to delay retirement beyond this age are small relative to the reduction in pension rights that results from the downward sloping life-cycle profile of earnings. Therefore, most workers who continue to work after age 65 face an implicit tax on doing so and many choose to leave the labor force at 65 to avoid this tax. Finally, Boldrin et al. (1997), Argimón et al. (2009), and Sánchez-Martín (2010) find that the probability of retiring at age 65 is independent of salary level, and our model economy replicates this stylized fact. Panel B of Figure 2 shows that retirement hazards at 65 are similar for the three educational groups, and that they are larger than 75 percent for all of them.

In Panel C of Figure 2 we report the shares of workers in the sum of workers, disabled people and retirees. We find that the age distribution of this ratio is almost identical in Spain and in the benchmark model economy.

Overall, we find these results very encouraging. A trustworthy answer to the questions that we ask in this paper requires a model economy that captures the key institutional and economic forces that affect the retirement decision. Our model economy replicates in great detail both the Spanish tax system and the rules of the Spanish public pension system. Moreover, our calibration procedure allows us to obtain an earnings process that allows us to replicate the earnings, income and wealth inequality observed in Spain, as we discuss below. And we have just shown that our model economy replicates many of the features of retirement behavior found in Spanish data. This result is particularly remarkable, since we did not target explicitly any of these retirement behavior facts in our calibration procedure.

Aggregates and Ratios. In Table 8 we report the macroeconomic aggregates and ratios in Spain and in our benchmark model economy for 2014. We find that our benchmark model economy replicates most of the Spanish targets reasonably well. The largest relative difference is in the consumption to output ratio which is approximately 4.5 percentage points lower in the model economy.

Inequality. In Table 9 we report the Gini indices and selected points of the Lorenz curves for earnings, income, and wealth in our model economy and in Spain. The statistics reported in bold face are our eight calibration targets. The source for the Spanish data on earnings, income and wealth is the 2004 Financial Survey of Spanish Families, as reported in Budría and Díaz-Giménez

[23]This effect can be reversed in the case of workers who expect to earn an exceptionally low salary for whatever reason. These workers face an implicit tax on continued work, since their low salaries reduce their pension rights and, therefore, their pensions.

Table 8: Macroeconomic Aggregates and Ratios in 2014 (%)

	C/Y^{*a}	K/Y^{*b}	l^c	T_y/Y^*	T_s/Y^*	P/Y^*
Spain	56.4	3.28	37.5	7.7	9.6	10.6
Model	51.9	3.29	37.1	7.9	9.4	10.8

[a]Variable Y^* denotes GDP at market prices.
[b]The target for K/Y^* is in model units and not in percentage terms.
[c]Variable l denotes the average share of their disposable time that the households allocate to the market.

(2006). The model economy statistics correspond to 2014.

Table 9: The Distributions of Earnings, Income, and Wealth*

		Bottom Tail			Quintiles					Top Tail		
	Gini	1	1–5	5–10	1st	2nd	3rd	4th	5th	10–5	5–1	1
The Earnings Distributions (%)												
Spain	**0.49**	0.0	0.7	1.2	**5.3**	10.9	16.2	23.3	**44.3**	10.9	11.5	5.6
Model	0.47	0.1	0.8	1.2	5.2	9.5	13.8	16.5	55.0	17.3	18.0	6.4
The Income Distributions (%)												
Spain	**0.42**	0.0	0.7	1.1	5.1	10.1	15.2	22.5	**47.1**	11.1	12.8	6.7
Model	0.44	0.1	0.9	1.6	6.3	9.5	13.9	17.7	52.5	14.4	18.2	6.8
The Wealth Distributions (%)												
Spain	**0.57**	–0.1	0.0	0.0	0.9	**6.6**	12.5	**20.6**	59.5	12.5	16.4	13.6
Model	0.56	0.0	0.0	0.0	1.0	7.0	13.7	20.9	57.3	15.3	21.7	6.1

*The source for the Spanish data of earnings, income, and wealth is the 2004 *Encuesta Financiera de las Familias Españolas* as reported in Budría and Díaz-Giménez (2006). The model economy statistics correspond to 2014. The statistics in bold face have been targeted in our calibration procedure.

We find that our model economy replicates the Spanish Gini indices of earnings, income and wealth reasonably well —the largest difference is only 0.02. Moreover it also comes close to replicating the Gini index of pensions. According to Conde-Ruiz and Profeta (2007), in 2000 this number was 0.32 in Spain and in our model economy it is 0.36 in our calibration year. Once again, this result can be interpreted as an overidentification condition, since we did not use it as a calibration target.

When we compare the various quantiles of the distributions, we find that the model economy households in the first four quintiles of the earnings distribution earn less than the Spanish households and that the households in the top quintile earn sizably more —their share of earnings is almost 11 percentage points higher than the Spanish share. In contrast, our model economy replicates the Spanish wealth distribution very closely. And, predictably, the income distribution is in between the other two —for instance, the share of income earned by the households in the top quintile of the model is almost 6 percentage points larger than the Spanish share, which is almost half way between 11 and -2.

When we look at the top tails of the distributions we find that the share of wealth owned by the top 1 percent of the wealth distribution is 7.5 percentage points higher in Spain. This disparity was to be expected, because it is a well-known result that overlapping generation model economies that abstract from bequests fail to account for the large shares of wealth owned by the very richest households in the data.[24]

3 Computation

To solve our model economy, we must choose the values of 50 parameters. As we have already mentioned, we the obtain the values of 31 of these parameters directly because they are functions of single targets. Another 4 parameters normalization conditions and 1 is obtained residually from the government budget constraint. This gives us a total of 36 parameters and leaves us with 14 to be determined. To do so, we solve a system of 14 non-linear equations.

The 14 parameters determined by this system are the following:

- Preferences: β and γ.
- Technology: δ.
- Stochastic process for labor productivity: $s(2), s(3), s_{11}, s_{12}, s_{21}, s_{22}, s_{32}$, and s_{33}.
- Pension system: ϕ and a_{14}.
- Fiscal policy: a_{10}.

To solve this system of equations we use a standard non-linear equation solver. Specifically, we use a modification of Powell's hybrid method, implemented in subroutine DNSQ from the SLATEC package.[25]

The DNSQ routine works as follows

1. Choose the weights that define the loss function that has to be minimized
2. Choose a vector of initial values for the 14 unknown parameters
3. Solve the model economy

[24] See Castañeda et al. (2003) for an elaboration of this argument.

[25] The benchmark model economy described in this paper is identical to the one described in Díaz-Giménez and Díaz-Saavedra (2017) except for an update in the calibration year and some other small changes. Therefore, we have used the same solution for the system of equations than in our previous paper, and we have not used DNSQ.

4. Update the vector of parameters

5. Iterate until no further improvements of the loss function can be found.

To solve the model economy, we proceed as follows:

1. We guess values for the interest rate, r, and for the effective labor input, N. Then, using the optimality conditions from the firm's maximization problem and the production function, we obtain the implied values for productive capital, K, output, Y, and the wage rate, w.

2. The value of output determines the values of the fiscal policy ratios, the values of the maximum and minimum pensions, and the pension grid, These variables, the tax rates already determined uniquely by single targets, and the remaining 3 government variables which are unknowns determine the government policy.

3. Given the factor prices, the government policy, the age-dependent probabilities of surviving, and the initial values of the parameters that describe preferences and the stochastic process for labor productivity, we solve the household's decision problem backwards and obtain household's optimal decisions.

4. We aggregate these optimal decisions and obtain the implied values for the government revenue items (tax collections and accidental bequests), pension payments, and the new values for K, N, r, w and Y.

5. Finally, we update N and r, and we iterate until convergence.

Once that the model economy is solved, DNSQ compares the relevant statistics of the model economy with the corresponding targets, and changes the initial values of the parameters to reduce the values of the loss function. This procedure continues until DNSQ cannot find further improvements of the loss function. At this point, the iteration stops and we have found a solution for the values of the 14 unknown parameters. Since the solutions to these very non-linear systems of equations are not guaranteed to exist and, when they do exist, they are not guaranteed to be unique, we try many different initial values for the 14 parameters and vectors of weights and we stop when we are convinced that we have found the best possible parameterization.

The system of equations is the following

$$
\begin{aligned}
0 &= 300 * ((C + T_c)/Y^* - 0.515) \\
0 &= 300 * (K/Y^* - 3.28) \\
0 &= 500 * (l - 0.375) \\
0 &= 500 * (P/Y^* - 0.103) \\
0 &= 30 * (T_s/Y^* - 0.101) \\
0 &= 30 * (T_y/Y^* - 0.0735) \\
0 &= 700 * (GY - 0.42) \\
0 &= 800 * (GE - 0.49) \\
0 &= 500 * (GW - 0.57) \\
0 &= 200 * (1QE - 0.053) \\
0 &= 100 * (5QY - 0.471) \\
0 &= 200 * (5QE - 0.443) \\
0 &= (2QW - 0.066) \\
0 &= (4QW - 0.206)
\end{aligned}
$$

and in Table 10 we report the initial values, the final values, and the weights that we have used to solve it and the errors that we have obtained.

Table 10: Initial Values, Final Values, Weights, and Errors.

Parameter	Initial Value	Final Value	Statistic	Weight (%)	Target	Result	Error (%)
δ	0.0653	0.0724	$(C + T_c)/Y^*$ (%)	300	0.515	0.514	-0.19
β	1.0459	1.0460	K/Y^*	300	3.28	3.28	0.00
γ	0.2900	0.2979	l (%)	500	37.50	37.62	0.32
ϕ	0.6308	0.8279	P/Y^* (%)	500	10.30	10.23	-0.68
a_{10}	0.2088	0.0672	T_s/Y^* (%)	30	10.13	10.10	-0.28
a_{14}	0.2373	0.2385	T_y/Y^* (%)	30	7.35	7.72	5.03
$s(2)$	2.4135	2.0856	GY	700	0.42	0.44	4.76
$s(3)$	5.6303	11.2892	GE	800	0.49	0.48	-2.04
s_{11}	0.9993	0.9417	GW	500	0.57	0.57	0.00
s_{12}	0.0006	0.0582	1QE (%)	200	5.30	5.20	-1.88
s_{21}	0.0007	0.0319	5QE (%)	200	44.30	55.70	25.73
s_{22}	0.9992	0.9680	5QY (%)	100	47.10	52.80	-12.1
s_{31}	0.0001	0.0000	2QW (%)	1	6.60	6.60	0.00
s_{32}	0.0007	0.0002	4QW (%)	1	20.60	20.50	-0.48

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